

HIGGS-GAUGE UNIFICATION WITHOUT TADPOLES

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In orbifold gauge theories localized tadpoles can be radiatively generated at the fixed points where $U(1)$ subgroups are conserved. If the Standard Model Higgs fields are identified with internal components of the bulk gauge fields (Higgs-gauge unification) in the presence of these tadpoles the Higgs mass becomes sensitive to the UV cutoff and electroweak symmetry breaking is spoiled. We find the general conditions, based on symmetry arguments, for the absence/presence of localized tadpoles in models with an arbitrary number of dimensions D . We show that in the class of orbifold compactifications based on T^{D-4}/\mathbb{Z}_N (D even, $N > 2$) tadpoles are always allowed, while on T^{D-4}/\mathbb{Z}_2 (arbitrary D) with fermions in arbitrary representations of the bulk gauge group tadpoles can only appear in $D = 6$ dimensions. We explicitly check this with one- and two-loops calculations.

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1 INTRODUCTION

The Standard Model (SM) of strong, weak and electromagnetic interactions is currently considered as an effective theory below a given cutoff Λ_{SM} . While this cutoff should be $\Lambda_{\text{EW}} \lesssim 1$ TeV for the stability of the Higgs mass under radiative corrections, present bounds from the non-observation of (dimension-six) four-fermion operators [1] push the SM cutoff towards $\Lambda \gtrsim 10$ TeV. This order of magnitude discrepancy between the cutoff Λ_{EW} (required for the stability of the electroweak symmetry breaking) and the cutoff Λ (implied by SM precision tests) already requires a certain amount of fine-tuning that is known as the little hierarchy problem [2].

Supersymmetry remains of course the best solution to the little (and grand) hierarchy problem, providing a SM cutoff of the order of the mass of supersymmetric partners $\Lambda_{\text{EW}} \sim M_{\text{SUSY}}$. The little hierarchy problem is naturally solved if R -parity is conserved: in this case supersymmetric virtual effects are loop suppressed and $\Lambda_{\text{EW}} \sim 4\pi\Lambda$. Nevertheless the minimal supersymmetric SM extension (the MSSM) is becoming very constrained by the LEP bounds from Higgs searches for radiative corrections to the Higgs quartic coupling increase only logarithmically with the scale M_{SUSY} which is a source of fine-tuning in the MSSM. It is thus compelling to propose possible alternative solutions to the little hierarchy problem that could fill the gap between the sub-TeV scale required for the stability of the electroweak symmetry breaking and the multi-TeV scale required by precision tests of the SM.

The possibility of TeV extra dimensions [3], suggested by string theories [4], has motivated an alternative solution to the little hierarchy problem called Higgs-gauge unification [5–12] in which the internal components of higher dimensional gauge bosons play the role of the SM Higgses that acquire a non-vanishing vacuum expectation value (VEV) through the Hosotani mechanism [13,14]. In this framework the Higgs mass in the bulk is protected from quadratic divergences by the higher-dimensional gauge theory and only finite corrections $\propto (1/R)^2$ (R is the compactification radius) that disappear in the $R \rightarrow \infty$ limit can appear. The SM cutoff is then identified with the inverse compactification radius $1/R$. On the other hand since the higher dimensional theory is non-renormalizable, it is in turn an effective theory with a cutoff Λ . The little hierarchy between $1/R$ and Λ is protected by the higher-dimensional gauge theory. Of course localized terms can be generated at the orbifold fixed points by quantum corrections [15–17] consistently with the symmetries of the theory [10,18]. A direct localized squared mass ($\sim \Lambda^2$) for the Higgs-gauge fields is forbidden by a shift symmetry at the orbifold fixed points: a remnant of the original bulk gauge symmetry [18].

It was however realized [10,11] that this approach could be jeopardized by the radiative generation of a localized tadpole in the cases where the bulk gauge group is broken at the orbifold fixed points into a group containing $U(1)$ factors, as e.g. the hypercharge $U(1)$. The tadpole corresponds to the field strength for internal space dimensions along the $U(1)$ -direction and contains in its non-abelian part a term quadratic in the Higgs-gauge

fields. In this way the tadpole quadratic divergence amounts to a quadratic divergence for the Higgs-gauge field squared mass and spoils the little hierarchy we wanted to solve. The existence of such tadpoles is a generic feature of orbifold compactifications in dimensions $D \geq 6$ and has been confirmed in six-dimensional orbifold field [12] and ten-dimensional string [19] theories. One way out [12] is that local tadpoles vanish globally and thus they would not spoil the four dimensional effective theory, although this requires a strong restriction on the bulk fermion content. Another possibility, that will be explored in this work, is that localized tadpoles be inconsistent with the symmetries of the theory in which case they will not be radiatively generated.

In this paper we will find the general conditions for the absence of localized tadpoles at the orbifold fixed points where $U(1)$ subgroups are conserved. They depend on the particular subgroup of the internal tangent space group $SO(D-4)$ which remains unbroken at the fixed point, and which constitutes a global invariance of the localized Lagrangian. In particular, if the internal rotation group acting on the component A_i is $SO(p)$ ($p > 2$), the absence of tadpoles involving that particular component is guaranteed, since only p -forms $F_{i_1 \dots i_p}$ can be linearly generated as $\epsilon^{i_1 \dots i_p} F_{i_1 \dots i_p}$, where $\epsilon^{i_1 \dots i_p}$ is the corresponding Levi-Civita tensor. Conversely if $p = 2$ the $U(1)$ tadpole can be generated through the gauge invariant form $\epsilon^{ij} F_{ij}$. The knowledge of such general conditions should enable us to find orbifold field theories with electroweak symmetry breaking triggered by Higgs-gauge fields where the Higgs mass is insensitive to the UV cutoff, although we will not attempt here to construct realistic tadpole-free models. Instead we will present a general class of such models based on \mathbb{Z}_2 orbifolds. In fact we will find that in the general class of T^{D-4}/\mathbb{Z}_2 orbifold compactifications tadpoles only appear in the case of $D = 6$ extra dimensions. For any other dimension the theory is tadpole-free. For $D \neq 6$ tadpoles are forbidden by the symmetries of the theory and we have explicitly checked this cancellation at the one- and two-loop levels. On the contrary we show that in the class of orbifold compactifications T^{D-4}/\mathbb{Z}_N (D even, $N > 2$) tadpoles are always allowed.

The rest of this paper is organized as follows. In Section 2 we present the general arguments based on symmetry considerations for the absence of localized tadpoles. In particular it is shown in Section 3 that the general class of compactifications based on T^{D-4}/\mathbb{Z}_2 for $D > 6$ fulfills the general requirements for the absence of localized tadpoles. In subsections 3.1 and 3.2 we show by explicit one- and two-loop calculations respectively that indeed no tadpoles appear in T^{D-4}/\mathbb{Z}_2 for $D \neq 6$. Section 4 contains some conclusive remarks while technical details about traces of Γ -matrices in arbitrary dimensions are presented in Appendix A.

2 GENERAL SYMMETRY ARGUMENTS

Given a compact d -dimensional ($d = D - 4$) manifold \mathcal{C} and a discrete symmetry group \mathbb{G} acting non-freely on it (i.e. with fixed points) we can define an orbifold by modding out \mathcal{C} by \mathbb{G} [20]. We will consider the case $\mathcal{C} = T^d$ where T^d is the d -dimensional torus obtained

by modding out \mathbb{R}^d by a d -dimensional lattice Λ^d : the orbifold group is then generated by a discrete subgroup of $SO(d)$ that acts crystallographically on the torus lattice and by discrete shifts that belong to the torus lattice. In particular if we define the coordinates as $x^M = (x^\mu, y^i)$, where x^μ are four-dimensional coordinates and y^i ($i = 1, \dots, d$) the torus coordinates, the action of $k \in \mathbb{G}$ on the torus is $k \cdot y = P_k y + u$ and the inverse is defined as $k^{-1} \cdot y = P_k^{-1} y - P_k^{-1} u$, where P_k is a discrete rotation in $SO(d)$ and $u \in \Lambda^d$; y and $k \cdot y$ are then identified on the orbifold. Since the orbifold group is acting non-freely on the torus there are fixed points characterized by $k \cdot y_f = y_f$. Any given fixed point y_f remains invariant under the action of a subgroup \mathbb{G}_f of the orbifold group.

The orbifold group acts on fields $\phi_{\mathcal{R}}$ transforming as an irreducible representation \mathcal{R} of the gauge group \mathcal{G} as

$$k \cdot \phi_{\mathcal{R}}(y) = \lambda_{\mathcal{R}}^k \otimes \mathcal{P}_{\sigma}^k \phi_{\mathcal{R}}(k^{-1} \cdot y) \quad (2.1)$$

where $\lambda_{\mathcal{R}}^k$ is acting on gauge and flavor indices and \mathcal{P}_{σ}^k , where σ refers to the field spin, on Lorentz indices. In particular one finds for scalar fields $\mathcal{P}_0^k = 1$ and for gauge fields $\mathcal{P}_1^k = P_k$ for a discrete rotation ($\mathcal{P}_1^k = 1$ for a lattice shift) that trivially follows from the invariance of the one-form $A = A_i dy^i$. Similarly the field strength $F = F_{ij} dy^i \wedge dy^j$ transforms as $\mathcal{P}_2^k F = P_k F P_k^T$. For fermions $\mathcal{P}_{\frac{1}{2}}^k$ can be derived requiring the invariance of the lagrangian under the orbifold action, as we will discuss in more detail later on. On the other side $\lambda_{\mathcal{R}}^k$ depends on the gauge structure and the gauge breaking of the orbifold action. $\lambda_{\mathcal{R}}^k$ and \mathcal{P}_{σ}^k are representations of the orbifold element k on the gauge and Lorentz groups, respectively.

In general the orbifold action breaks the gauge group in the bulk $\mathcal{G} = \{T^A\}$ to a subgroup $\mathcal{H}_f = \{T^{a_f}\}$ at the fixed point y_f . The orbifold group element k acts as a Lie algebra automorphism $T^A \rightarrow \Lambda_k^{AB} T^B$ that can be represented as a group conjugation $T^A \rightarrow g_k T^A g_k^{-1}$ in case of an inner automorphism (as we will consider here), where g_k is a representation on \mathcal{G} of the orbifold group element k . A convenient basis to write the element g_k is the Weyl-Cartan basis of the Lie algebra $\{T^A\} = \{E^{\alpha}, H^I\}$, where $\vec{\alpha}$ is the $\text{rank}(\mathcal{G})$ -dimensional root associated to the generator E^{α} . In this case the gauge group breaking is characterized by the vector \vec{v}^k as

$$\lambda_{\mathcal{R}}^k = \exp(-2\pi i \vec{v}^k \cdot \vec{H}_{\mathcal{R}}) \quad (2.2)$$

and indeed one can identify $g^k = \lambda_{Adj}^k$ and the Lie algebra automorphism is given by $\Lambda_k = \text{diag}[\delta_{IJ}, \exp(-2\pi i \vec{v}^k \cdot \vec{\alpha}) \delta_{\alpha\beta}]$. The group elements $\lambda_{\mathcal{R}}^k$ satisfy the automorphism condition $\lambda_{\mathcal{R}}^k T_{\mathcal{R}}^A \lambda_{\mathcal{R}}^{-k} = \Lambda^{AB} T_{\mathcal{R}}^B$, from where it follows that the subgroup \mathcal{H}_f left invariant by the orbifold elements $k \in \mathbb{G}_f$ is defined by the generators that commute with $\lambda_{\mathcal{R}}^k$, i.e. $[\lambda_{\mathcal{R}}^k, T_{\mathcal{R}}^{a_f}] = 0$. Of course the latter condition must be satisfied by any irreducible representation \mathcal{R} of \mathcal{G} .

In the same way as the orbifold action breaks the bulk gauge group \mathcal{G} to a subgroup \mathcal{H}_f , such that $[\lambda^k, \mathcal{H}_f] = 0$, it also breaks the internal rotation group $SO(d)$ into a subgroup \mathcal{O}_f at the orbifold fixed point y_f . In fact in compactifications to a smooth d -dimensional Riemannian manifold (with positive signature) the orthogonal transformations acting on

the tangent space at a given point form the group $SO(d)$ [21]. At the orbifold fixed point y_f a further compatibility condition between the orbifold action and the internal rotations is required. In particular, if the given fixed point y_f is left invariant by the orbifold subgroup \mathbb{G}_f , only \mathbb{G}_f -invariant operators $\Phi_{\mathcal{R},\sigma}$ (either invariant fields, products of non-invariant fields or derivatives of fields) couple to y_f , i.e.

$$k \cdot \Phi_{\mathcal{R},\sigma}(y_f) = \Phi_{\mathcal{R},\sigma}(y_f) . \quad (2.3)$$

Acting on $\Phi_{\mathcal{R},\sigma}$ with an internal rotation we get a transformed operator that should also be \mathbb{G}_f -invariant. This means, using Eq. (2.1), that the subgroup \mathcal{O}_f is spanned by the generators of $SO(d)$ that commute with \mathcal{P}_σ^k , i.e. they satisfy the condition

$$[\mathcal{O}_f, \mathcal{P}_\sigma^k] = 0 \quad (2.4)$$

for $k \in \mathbb{G}_f$ and arbitrary values of σ . In particular in the presence of gauge fields $A_M = (A_\mu, A_i)$ an invariant operator can be F_{ij} with $\mathcal{R} = Adj$ and $\sigma = 2$. The internal components A_i transform under the action of the orbifold element $k \in \mathbb{G}_f$ as the discrete rotation P_k . At the orbifold fixed point y_f only the subgroup $\mathcal{O}_f \subseteq SO(d)$ survives and the vector representation A_i of $SO(d)$ breaks into irreducible representations of \mathcal{O}_f .

We now consider a gauge theory coupled to fermions. The lagrangian of the orbifold theory is the sum of a bulk D -dimensional lagrangian \mathcal{L}_D and four-dimensional lagrangians \mathcal{L}_f localized at the orbifold fixed points $y = y_f$ as

$$\mathcal{L} = \mathcal{L}_D + \sum_f \delta^{(d)}(y - y_f) \mathcal{L}_f . \quad (2.5)$$

The bulk lagrangian is given by

$$\mathcal{L}_D = -\frac{1}{4} F_{MN}^A F^{AMN} + i \bar{\Psi} \Gamma_D^M D_M \Psi \quad (2.6)$$

with $F_{MN}^A = \partial_M A_N^A - \partial_N A_M^A - gf^{ABC} A_M^B A_N^C$, $D_M = \partial_M - ig A_M^A T^A$ and where Γ_D^M are the Γ -matrices corresponding to a D -dimensional space-time that are defined in appendix A. The four-dimensional lagrangians \mathcal{L}_f must be invariant under the usual four-dimensional symmetries: gauge $[\mathcal{H}_f]$ and Lorentz $[SO(3,1)]$ symmetries. On top of that they must be invariant under the action of the orbifold group and the remnant symmetries left out by the orbifold compactification: the remnant gauge symmetry and the internal rotation group \mathcal{O}_f . We will now briefly comment about the two latter symmetries. Notice that they are global symmetries of the Lagrangian \mathcal{L}_f .

The invariance under bulk (infinitesimal) gauge transformations

$$\delta_\xi A_M^A = \frac{1}{g} \partial_M \xi^A - f^{ABC} \xi^B A_M^C, \quad \delta_\xi \Psi = i \xi^A T^A \Psi \quad (2.7)$$

translates, when applied to the orbifold fixed points y_f , into the four-dimensional gauge symmetry ³ $\mathcal{H} = \{T^a\}$ that applies to the four-dimensional gauge fields A_μ^a which are

³From here on we will remove for simplicity the subscript “f” from the gauge group and the corresponding generators.

also invariant under the orbifold action and leads to the usual gauge invariance under \mathcal{H} -transformations $\delta_\xi A_\mu^a = \partial_\mu \xi^a / g - f^{abc} \xi^b A_\mu^c$. By localizing the transformations (2.7) at the orbifold fixed point y_f and keeping the orbifold invariant terms one can define an infinite set of transformations (remnant of the bulk gauge invariance) induced by derivatives of ξ^A that we can call \mathcal{K} -transformations [10, 18]. Then only \mathcal{H} and \mathcal{K} -invariant quantities are allowed at the orbifold fixed points. For instance if the gauge field $A_i^{\hat{a}}$ is invariant under the orbifold action, where $T^{\hat{a}} \in \mathcal{G}/\mathcal{H}$, the remnant “shift” symmetry $\delta_\xi A_i^{\hat{a}} = \partial_i \xi^{\hat{a}} / g + \dots$ prevents the corresponding zero mode from acquiring a mass localized at the orbifold fixed point.

Gauge fields along the internal dimensions A_i^a are scalars in the adjoint representation of the group \mathcal{H} that transform under the orbifold action as $P_k A$; they are not orbifold invariant and cannot couple to fixed points. However the corresponding field strength F_{ij}^a transforms under the orbifold action as $k \cdot F_{ij}^a = (P_k)_i^m (P_k)_j^n F_{mn}^a$ and, depending on the orbifold, some components F_{ij}^a can be orbifold invariant. On the other hand gauge fields along internal dimensions and components in the coset $\mathcal{G}/\mathcal{H} = \{T^{\hat{a}}\}$ transform under the orbifold group as $k \cdot A_i^{\hat{a}} = (P_k)_i^j \Lambda_k^{\hat{a}B} A_j^B$: some components $A_i^{\hat{a}}$ can be orbifold invariant. Under these circumstances if $\mathcal{H} \supseteq U(1)$ the “tadpole” F_{ij}^α where α is the $U(1)$ quantum number

$$F_{ij}^\alpha = \partial_i A_j^\alpha - \partial_j A_i^\alpha - g f^{\alpha\hat{b}\hat{c}} A_i^{\hat{b}} A_j^{\hat{c}} \quad (2.8)$$

is \mathcal{H} and \mathcal{K} -invariant as well as orbifold invariant. Notice that the (orbifold invariant part of the) last term can appear as a mass term for zero modes.

In summary if F_{ij}^α appears localized at orbifold fixed points it can contain a zero mode mass term that can destabilize (break) the gauge theory ⁴. For orbifold compactifications breaking \mathcal{G} into $U(1)$ subgroups at the fixed points the existence of gauge and orbifold-invariant field strengths F_{ij}^α that can appear in localized lagrangians is a generic feature in any model. The further requirement for the tadpole appearance is that internal rotation invariance be conserved at the fixed point.

We have previously seen that the vector representation A_i of $SO(d)$ breaks into irreducible representations of the internal rotation group $\mathcal{O}_f \subseteq SO(d)$ that commutes with \mathcal{P}_σ^k . In particular if the rotation subgroup acting on the (i, j) -indices is $SO(2)$ then $\epsilon^{ij} F_{ij}^\alpha$, where ϵ^{ij} is the Levi-Civita tensor, is invariant under \mathcal{O}_f . On the other hand if the rotation subgroup acting on the (i, j) -indices is $SO(p)$ ($p > 2$) then the Levi-Civita tensor would be $\epsilon^{i_1 i_2 \dots i_p}$ and only invariants constructed using p-forms would be allowed. In other words a sufficient condition for the absence of localized tadpoles is that the smallest internal subgroup factor be $SO(p)$ ($p > 2$).

We can exemplify the different possibilities by considering a class of $\mathbb{G} = \mathbb{Z}_N$ orbifolds

⁴We assume here that $U(1)$ is not contained in the bulk group \mathcal{G} . Otherwise the non-abelian term in (2.8) does not exist and the tadpole is harmless as far as the electroweak breaking is concerned.

for even d where the generator P_N of the orbifold group is defined by

$$P_N = \prod_{i=1}^{d/2} e^{2\pi i \frac{k_i}{N} J_{2i-1,2i}} \quad (2.9)$$

where k_i are integer numbers ($0 < k_i < N$) and $J_{2i-1,2i}$ is the generator of a rotation with angle $2\pi \frac{k_i}{N}$ in the plane (y_{2i-1}, y_{2i}) . All orbifold elements are defined by $P_k = P_N^k$ ($k = 1, \dots, N-1$) and satisfy the condition $P_N^N = 1$. The generator of rotations in the (y_{2i-1}, y_{2i}) -plane can be written as $J_{2i-1,2i} = \text{diag}(0, \dots, \sigma^2, \dots, 0)$ where the Pauli matrix σ^2 is in the i -th two-by-two block. The generator P_N can be written as $P_N = \text{diag}(R_1, \dots, R_{d/2})$ where the discrete rotation in the (y_{2i-1}, y_{2i}) -plane is defined as

$$R_i = \begin{pmatrix} c_i & s_i \\ -s_i & c_i \end{pmatrix} \quad (2.10)$$

with $c_i = \cos(2\pi k_i/N)$, $s_i = \sin(2\pi k_i/N)$.

Let y_f be a fixed point that is left invariant under the orbifold subgroup $\mathbb{G}_f = \mathbb{Z}_{N_f}$ where $N_f \leq N$. We now define the internal rotation group \mathcal{O}_f as the subgroup of $SO(d)$ that commutes with the generator of the orbifold \mathbb{Z}_{N_f} , P_{N_f} as given by Eq. (2.9) with N replaced by N_f . In general, if $N_f > 2$ it is trivially provided by the tensor product:

$$\mathcal{O}_f = \bigotimes_{i=1}^{d/2} SO(2)_i \quad (2.11)$$

where $SO(2)_i$ is the $SO(2) \subseteq SO(d)$ that acts on the (y_{2i-1}, y_{2i}) -subspace. In every such subspace the metric is δ_{IJ} and the Levi-Civita (antisymmetric) tensor ϵ^{IJ} ($I, J = 2i-1, 2i$, $i = 1, \dots, d/2$) such that we expect the tadpoles appearance at the fixed points y_f as

$$\sum_{i=1}^{d/2} \mathcal{C}_i \sum_{I, J=2i-1}^{2i} \epsilon^{IJ} F_{IJ}^\alpha \delta^{(d/2)}(y - y_f). \quad (2.12)$$

If $N_f = 2$ then the generator of the orbifold subgroup $\mathbb{G}_f = \mathbb{Z}_2$ is the inversion $P = -\mathbf{1}$ that obviously commutes with all generators of $SO(d)$ and $\mathcal{O}_f = SO(d)$. In this case the Levi-Civita tensor is $\epsilon^{i_1 \dots i_d}$ and only a d -form can be generated linearly in the localized lagrangian. Therefore tadpoles are only expected in the case of $d = 2$ ($D = 6$).

The last comments also apply to the case of \mathbb{Z}_2 orbifolds of arbitrary dimensions (even or odd) since in that case the orbifold generator is always $P = -\mathbf{1}$ and the internal rotation group that commutes with P is $\mathcal{O}_f = SO(d)$ for all the fixed points. Again tadpoles are only expected for $D = 6$ dimensions while they should not appear for $D > 6$ ⁵.

Since every operator in \mathcal{L}_f that is consistent with all the symmetries should be radiatively generated by loop effects from matter in the bulk (unless it is protected by some other –accidental– symmetry) explicit calculations of the tadpole in orbifold gauge theories should confirm the appearance (or absence) of them in agreement with the above symmetry arguments. In the rest of this paper we will explicitly present the case of \mathbb{Z}_2 -orbifolds in arbitrary dimensions.

⁵Notice that tadpoles vanish identically for the case $D = 5$.

3 \mathbb{Z}_2 ORBIFOLDS IN ARBITRARY DIMENSION

We consider in this section the case of a gauge theory coupled to fermions in $D > 4$ dimensions. The bulk lagrangian is given in Eq. (2.6) and the extra dimensions are compactified on the orbifold T^d/\mathbb{Z}_2 , with the action of \mathbb{Z}_2 defined by the identification $y^i = -y^i$. The orbifold group \mathbb{Z}_2 has a single element (apart from the identity) i.e. $P = -\mathbf{1}$.

The parity assignment for gauge fields is characterized by the diagonal matrix $\Lambda = \text{diag}(\eta^A)$ with $\eta^A = \pm 1$. It can then be written as:

$$A_M^A(x^\mu, -y^i) = \eta^A \alpha_M A_M^A(x^\mu, y^i) , \quad (3.1)$$

with $\alpha_\mu = +1$, $\alpha_i = -1$, $\eta^a = +1$ and $\eta^{\hat{a}} = -1$. Here a corresponds to the unbroken generators of the gauge group, while \hat{a} corresponds to the broken ones. The only conditions we need for the Yang-Mills term to be invariant under this \mathbb{Z}_2 action is the automorphism condition [17, 22]

$$\eta^A \eta^B \eta^C = 1 \text{ for } f^{ABC} \neq 0 . \quad (3.2)$$

The action of \mathbb{Z}_2 on fermions in representation \mathcal{R} of the gauge group \mathcal{G} is

$$\Psi_{\mathcal{R}}(x^\mu, -y^i) = \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \Psi_{\mathcal{R}}(x^\mu, y^i) , \quad (3.3)$$

where $\lambda_{\mathcal{R}}$ acts on representation indices and $\mathcal{P}_{\frac{1}{2}}$ on spinor indices. From the requirement that the kinetic term for $\Psi_{\mathcal{R}}$ is invariant under the parity action we obtain the following constraint on $\mathcal{P}_{\frac{1}{2}}$:

$$\Gamma_D^0 \mathcal{P}_{\frac{1}{2}} \Gamma_D^0 \Gamma_D^M = \alpha_M \Gamma_D^M \mathcal{P}_{\frac{1}{2}} , \quad (3.4)$$

which translates into two possible different conditions:

$$\begin{aligned} \text{if } [\Gamma_D^0, \mathcal{P}_{\frac{1}{2}}] = 0 & \Rightarrow \begin{cases} [\Gamma_D^\mu, \mathcal{P}_{\frac{1}{2}}] = 0 \\ \{\Gamma_D^i, \mathcal{P}_{\frac{1}{2}}\} = 0 \end{cases} & (a) \\ \text{if } \{\Gamma_D^0, \mathcal{P}_{\frac{1}{2}}\} = 0 & \Rightarrow \begin{cases} \{\Gamma_D^\mu, \mathcal{P}_{\frac{1}{2}}\} = 0 \\ [\Gamma_D^i, \mathcal{P}_{\frac{1}{2}}] = 0 . \end{cases} & (b) \end{aligned} \quad (3.5)$$

All this is valid for every D . It can be shown that for D even there is a solution in both cases, while for D odd only (b) has a solution. These are precisely:

$$\begin{aligned} D \text{ even} & \begin{cases} (a) \Rightarrow \mathcal{P}_{\frac{1}{2}} = \beta_D \Gamma_D^5 \dots \Gamma_D^D \\ (b) \Rightarrow \mathcal{P}'_{\frac{1}{2}} = \beta_D \Gamma_D^5 \dots \Gamma_D^D \Gamma_D^{D+1} = \mathcal{P}_{\frac{1}{2}} \Gamma_D^{D+1} \end{cases} \\ D \text{ odd} & \begin{cases} (a) \Rightarrow \text{no solution} \\ (b) \Rightarrow \mathcal{P}'_{\frac{1}{2}} = -i \beta_{D-1} \Gamma_D^5 \dots \Gamma_D^D , \end{cases} \end{aligned} \quad (3.6)$$

where β_D is such that $\mathcal{P}_{\frac{1}{2}}^2 = \mathbf{1}$ (and therefore $\mathcal{P}'_{\frac{1}{2}}^2 = \mathbf{1}$). Up to now we have considered only the kinetic term; from the requirement that also the interaction term is invariant

under the parity transformation, we obtain the following conditions on $\lambda_{\mathcal{R}}$ [17]:

$$\eta^A \lambda_{\mathcal{R}} T^A \lambda_{\mathcal{R}} = T^A \quad \Leftrightarrow \quad \begin{cases} [\lambda_{\mathcal{R}}, T^a] = 0 \\ \{\lambda_{\mathcal{R}}, T^{\hat{a}}\} = 0 \end{cases} \quad (3.7)$$

These requirements are valid for any D .

3.1 ONE-LOOP CALCULATION OF TADPOLES

Since we want to compute radiative corrections, we must define the Feynman rules. It is well known that in an orbifold field theory all the information concerning the orbifold can be inserted in the propagator of the KK-modes of the fields, leaving the vertices momentum-conserving [15]. In this picture the propagator of the \vec{m} -mode ($\vec{m} = (m_1, \dots, m_d)$) of an arbitrary field Φ (a gauge boson A_M , a ghost field c or a fermion field Ψ) in the D dimensional space compactified on T^d/\mathbb{Z}_2 can be written in terms of the propagator of a field living in the torus T^d in the following way:

$$\langle \Phi^{\vec{m}'} \bar{\Phi}^{\vec{m}} \rangle = \frac{1}{2} G^{\Phi}(p_{\mu}, p_i) (\delta_{\vec{m}' - \vec{m}} \pm \mathcal{P}_{\Phi} \delta_{\vec{m}' + \vec{m}}) \quad (3.8)$$

Here $G^{\Phi}(p_{\mu}, p_i)$ is the propagator of Φ in a D dimensional Minkowski space where the torus periodicity conditions $p_i = m_i/R$ are imposed ⁶, \mathcal{P}_{Φ} is the parity action defined in Eqs. (3.1)-(3.3) and the “ $-$ ” sign only applies to fermions if $\mathcal{P}_{\frac{1}{2}}$ anticommutes with Γ_D^0 . The propagators in a D dimensional Minkowski space-time and in the Feynman gauge are the following:

$$\begin{aligned} G^A(p_{\mu}, p_i) &= -i \frac{\delta^{BC}}{p^2} g_{MN} \\ G^c(p_{\mu}, p_i) &= -i \frac{\delta^{BC}}{p^2} \\ G^{\Psi}(p_{\mu}, p_i) &= i \frac{\Gamma_D^M p_M}{p^2}, \end{aligned} \quad (3.9)$$

where $p^2 \equiv p^M p_M$ is the D -dimensional momentum. The vertices are given by (2.6) and by the gauge fixing and ghost lagrangian. In the Feynman gauge the latter is given by

$$\mathcal{L}_{\text{gf}} + \mathcal{L}_{FP} = -\frac{1}{2} (g^{MN} \partial_M A_N^A)^2 - \text{tr } \bar{c} \partial^M D_M c. \quad (3.10)$$

3.1.1 FERMION CONTRIBUTION

We begin by considering D even in which case fermions can be chiral. The one-loop fermion contribution to the tadpole $\partial_i A_j^a$ is given by the diagram of Fig. 1 where the fermionic line contains a projector \mathcal{P}_{Ψ} coming from the expansion in Eq. (3.8).

⁶We are assuming a common compactification radius R for all internal dimensions and orthogonal lattice vectors.

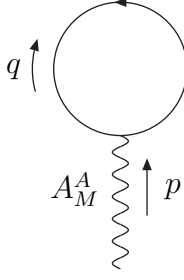


Figure 1: One-loop fermion contribution to the tadpole

The fermion contribution to the tadpole turns out to be proportional to:

$$tr \left\{ \frac{\Gamma_D^N q_N}{q^2} \left(\delta_{\vec{m}' - \vec{m}} \pm \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{m}' + \vec{m}} \right) \delta_{\vec{m}' - \vec{m} + \vec{l}} (\mathbf{1} \pm \Gamma_D^{D+1}) \Gamma_D^M T_{\mathcal{R}}^A \right\} . \quad (3.11)$$

The term proportional to $\delta_{\vec{m}' - \vec{m}}$ which corresponds to the bulk contribution is identically zero since it contains $tr\{T_{\mathcal{R}}^A\}$ which vanishes. We consider then the brane contribution, which can be factorized as follows:

$$tr \left\{ \Gamma_D^N \mathcal{P}_{\frac{1}{2}} (\mathbf{1} \pm \Gamma_D^{D+1}) \Gamma_D^M \right\} tr \left\{ \lambda_{\mathcal{R}} T_{\mathcal{R}}^A \right\} \frac{q_N}{q^2} . \quad (3.12)$$

By simply considering the trace relative to the Γ -matrices, it is possible to show that Eq. (3.12) can be non-zero only for $D = 6$. Indeed this term can be written as

$$tr \left\{ \Gamma_D^N \mathcal{P}_{\frac{1}{2}} (\mathbf{1} \pm \Gamma_D^{D+1}) \Gamma_D^M \right\} = tr \left\{ \mathcal{P}_{\frac{1}{2}} \Gamma_D^M \Gamma_D^N \right\} \pm tr \left\{ \mathcal{P}_{\frac{1}{2}} \Gamma_D^{D+1} \Gamma_D^M \Gamma_D^N \right\} , \quad (3.13)$$

where $\mathcal{P}_{\frac{1}{2}}$ is one of the two solutions of Eq. (3.6). If $\mathcal{P}_{\frac{1}{2}} \propto \Gamma_D^5 \dots \Gamma_D^D$ (case *a*), using the rules on the trace enumerated in the appendix, it can be shown that only the first term of the right-hand side of Eq. (3.13) can be different from zero and this only happens for $D = 6$. In this particular case the result is:

$$tr \left\{ \Gamma_D^5 \Gamma_D^6 \Gamma_D^M \Gamma_D^N \right\} = 8(g^{5N} g^{6M} - g^{5M} g^{6N}) = -8g^{Mi} g^{Nj} \epsilon_{ij} . \quad (3.14)$$

In case (*b*), where $\mathcal{P}'_{\frac{1}{2}} = \mathcal{P}_{\frac{1}{2}} \Gamma_D^{D+1}$, the traces we have to evaluate are the same but inverted. This means that only the second term of the right-hand side of Eq. (3.13) can be different from zero, the result being the one quoted above. The only noticeable thing is that while in case (*a*) the contribution to the tadpole was chiral-independent, in case (*b*) it is chiral-dependent. This means that in six dimensions if fermions transform under parity according to $\mathcal{P}_{\frac{1}{2}}$ we have a non-vanishing tadpole both with Dirac and Weyl fermions. On the contrary, if they transform with $\mathcal{P}'_{\frac{1}{2}}$, the tadpole can be zero if we are in presence of Dirac fermions, since fermions with different chirality give opposite contributions. This is a consequence of an extra parity symmetry of the theory which inverts separately the internal coordinates: the term $\epsilon^{ij} F_{ij}$ is odd under this symmetry and therefore it cannot be generated [10–12].

As already discussed in Ref. [10], there is a close relation between the tadpole generated by the diagram of Fig. 1 and the mixed $U(1)$ -gravitational anomaly in a $6D$ theory. In fact fermions with different $6D$ chiralities contribute with the same sign to the tadpole and with opposite sign to the anomaly or viceversa, depending on the particular $\mathcal{P}_{\frac{1}{2}}$ considered. As a consequence, starting with Dirac fermions, a vanishing anomaly implies a non-vanishing tadpole and viceversa. The cancellation conditions are the same for the tadpole and the anomaly only when dealing with chiral fermions.

We now move to the case of D odd. Here chirality does not exist so the fermion contribution is proportional to:

$$tr \left\{ \frac{\Gamma_D^N q_N}{q^2} \left(\delta_{\vec{m}' - \vec{m}} \pm \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{m}' + \vec{m}} \right) \delta_{\vec{m}' - \vec{m} + \vec{l}} \Gamma_D^M T_{\mathcal{R}}^A \right\} . \quad (3.15)$$

This means that the only trace we have to evaluate is

$$tr \left\{ \mathcal{P}_{\frac{1}{2}} \Gamma_D^M \Gamma_D^N \right\} . \quad (3.16)$$

Now $\mathcal{P}_{\frac{1}{2}}$ is unique and proportional to $\Gamma_D^5 \dots \Gamma_D^D$, where the $\Gamma_D^D = \Gamma_{D-1}^{(D-1)+1}$. It is not difficult to show that this contribution is always zero for any D odd.

3.1.2 GAUGE AND GHOST CONTRIBUTION

The gauge and ghost one-loop contributions to the tadpole $\partial_i A_j$ are given by the diagrams in Fig. 2. They do not contribute to the tadpole for any dimension D . For $D = 6$ the

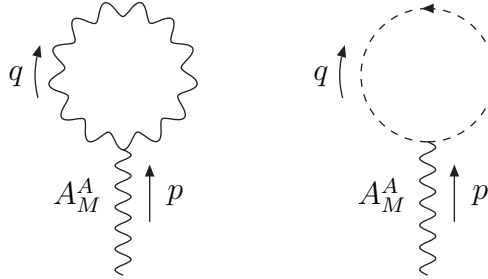


Figure 2: One-loop gauge and ghost contributions to the tadpole

proof was done in Ref. [10] although the cancellation is more general and could be applied to any dimension. Indeed the gauge contribution is proportional to:

$$\delta^{BC} f^{ABC} \quad (3.17)$$

and this is clearly zero. The ghost contribution is also proportional to this trace, so we can conclude that we do not have any contribution from the gauge sector for any number of dimensions D . This is a consequence of the same parity symmetry we previously discussed for fermions. While there its existence depends on the particular $\mathcal{P}_{\frac{1}{2}}$ considered, in the gauge sector it always subsists, forbidding the appearance of the tadpole [11, 12].

3.2 TWO-LOOP CALCULATION

The cancellation of two-loop diagrams involving only gauge and ghost fields was done in Ref. [10] for the case of $D = 6$ dimensions. Since the proof given there also applies for any dimension we will skip it here. Two-loop diagrams involving fermion loops (where fermions belong to the representation \mathcal{R} of \mathcal{G}) are shown in Fig. 3 where all momenta are D -dimensional and the external four-momentum is zero, i.e. $p = (0, p^i)$.

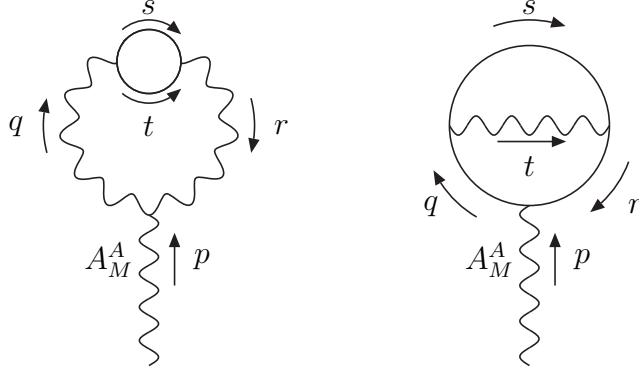


Figure 3: Two-loop tadpole diagrams with fermions

The contribution of localized tadpoles corresponds to the given diagrams with orbifold projections \mathcal{P}_Φ acting on some (or all) internal lines. In particular there is no localized contribution only in the obvious case without any orbifold projections and in the case in which there are two orbifold projections on the legs connected to the external one. Anyway we will show that also in these cases the (non-localized) result is zero as it should. An orbifold projection on the gauge boson propagator in Eq. (3.9) amounts to the insertion of $\eta^B \alpha_M$ and one on the fermion propagator amounts to inserting $\lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}}$. Since the orbifold projection is twisting the arrow of internal momenta we will call an internal line with an orbifold projection a “twisted” line.

Let us first discuss the diagram on the left of Fig. 3. It is proportional to:

$$\begin{aligned} & \sum_{\vec{q}, \vec{s}, \vec{q}', \vec{s}'} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 s}{(2\pi)^4} \frac{1}{q^2} \frac{1}{s^2} \frac{1}{t^2} \frac{1}{r^2} \text{tr} \left\{ (\delta_{\vec{q}-\vec{q}'} + \eta^B \alpha_N \delta_{\vec{q}+\vec{q}'}) \right. \\ & \Gamma_D^N T_{\mathcal{R}}^B \delta_{\vec{q}'-\vec{s}-\vec{t}} \not{s} \left(\delta_{\vec{s}-\vec{s}'} + \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{s}+\vec{s}'} \right) \Gamma_D^R T_{\mathcal{R}}^C \delta_{\vec{s}'+\vec{t}'-\vec{r}'} \not{t} \\ & \left(\delta_{\vec{t}-\vec{t}'} + \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{t}+\vec{t}'} \right) (\delta_{\vec{r}-\vec{r}'} + \eta^C \alpha_R \delta_{\vec{r}+\vec{r}'}) f^{ABC} \\ & \left. [(r+q)_M g_{NR} - (q+p)_R g_{MN} + (p-r)_N g_{MR}] \delta_{\vec{p}-\vec{q}+\vec{r}} \right\}, \end{aligned} \quad (3.18)$$

where $t^\mu = q^\mu - s^\mu$ and $r^\mu = q^\mu - p^\mu = q^\mu$ and we skip the insertion of possible chirality factors $(\mathbf{1} \pm \Gamma_D^{D+1})$ for the case of chiral fermions in even dimensions. We see that there are sixteen different diagrams which correspond to the different possibilities for inserting projections, some of them differing only by permutations of twisted lines. These sixteen

diagrams can be divided into three groups according to the number of twisted fermions: there are four diagrams with no twisted fermions, eight with one twisted fermion and four with two.

Diagrams with no twisted fermions are proportional to

$$f^{ABC} \text{tr} (T_{\mathcal{R}}^B T_{\mathcal{R}}^C) = f^{ABC} \delta^{BC} C_2(\mathcal{R}) = 0 \quad (3.19)$$

and so they cancel for any dimension. The presence or not of twisted bosons does not affect this result, since it only consists in multiplying this by $\eta^{B,C}$.

Diagrams with two twisted fermions are proportional to

$$f^{ABC} \text{tr} (T_{\mathcal{R}}^B \lambda_{\mathcal{R}} T_{\mathcal{R}}^C \lambda_{\mathcal{R}}) = f^{ABC} \eta^C \delta^{BC} C_2(\mathcal{R}) = 0 \quad (3.20)$$

and so they equally cancel for any dimension.

Therefore we only have to compute diagrams with one twisted fermion. Considering all the possible permutations of twists we have eight diagrams, with zero, one or two twisted gauge bosons. The diagrams in which the momentum s is twisted are proportional to

$$\text{tr} \left\{ \Gamma_D^N \not{\mathcal{P}}_{\frac{1}{2}} \Gamma_D^R \not{\mathcal{P}}_{\frac{1}{2}} \right\} [(r+q)_M g_{NR} - (q+p)_R g_{MN} + (p-r)_N g_{MR}] \quad (3.21)$$

and the presence of twisted gauge bosons only amounts to the insertion of $\eta^{B,C}$ and, obviously, to a change in the flux of momenta. Now we evaluate the above expression in the case of $M = i$, which corresponds to have a gauge boson A_i on the external leg. Indeed the only quadratically divergent term that can be generated at the orbifold fixed points consistently with the gauge symmetry is F_{ij}^α , so we can concentrate ourselves in isolating contributions to $\partial_j A_i$ that precisely correspond to $M = i$ and an external momentum p^j . We obtain:

$$\begin{aligned} & \text{tr} \left\{ \Gamma_D^N \Gamma_D^S \not{\mathcal{P}}_{\frac{1}{2}} \Gamma_D^R \Gamma_D^T \right\} s_S t_T (r+q)_i g_{NR} - \\ & \text{tr} \left\{ \Gamma_D^i \Gamma_D^S \not{\mathcal{P}}_{\frac{1}{2}} \Gamma_D^R \Gamma_D^T \right\} s_S t_T (q+p)_R + \\ & \text{tr} \left\{ \Gamma_D^N \Gamma_D^S \not{\mathcal{P}}_{\frac{1}{2}} \Gamma_D^i \Gamma_D^T \right\} s_S t_T (p-r)_N. \end{aligned} \quad (3.22)$$

We first consider the second and the third terms. We see that for D odd these are zero, since $\not{\mathcal{P}}_{\frac{1}{2}}$ contains a $\Gamma_{(D-1)}^{(D-1)+1}$ and the trace could be non zero only if all the four Γ -matrices were of μ -type. For D even fermions can be chiral in which case there should be a corresponding insertion of $(\mathbf{1} \pm \Gamma_D^{D+1})$ in the trace over Γ -matrices. The terms containing Γ_D^{D+1} vanish (using the same argument as that of odd dimensions) and the remaining terms are the ones present in Eq. (3.22). The argument is reversed if we use $\not{\mathcal{P}}_{\frac{1}{2}}'$ but the final result will remain unchanged so that without loss of generality we will consider the projection $\mathcal{P}_{\frac{1}{2}}$. For $D \geq 10$ the trace is zero, for the same reason it was zero in the one-loop case for $\bar{D} \geq 8$. So we have to see what happens for $D = 8$ (we do not need to consider the case with $D = 6$ since in this case there are contributions already

at one loop). In this case the trace turns out to be proportional to ϵ^{ijkl} (modulo index permutations), which is the completely antisymmetric tensor. Now we have to look at the momenta: if we calculate the fluxes in all the considered cases we find that these are symmetric in the indices $ijkl$, so that also for $D = 8$ the contribution of the second and the third terms of Eq. (3.22) vanishes. Now we move to the first term. This can be rewritten in the following form:

$$(2 - D) \operatorname{tr} \left\{ \Gamma_D^S \mathcal{P}_{\frac{1}{2}} \Gamma_D^T \right\} s_S t_T (r + q)_i. \quad (3.23)$$

We immediately see that the trace we have to compute is the same as in the one-loop case, so we can conclude that we can have a contribution only for $D = 6$. Up to now we have computed diagrams in which the twisted momentum is s . The computation of diagrams in which t is twisted is analogous and leads to the same result.

Now we consider the diagram on the right of Fig. 3. It is proportional to:

$$\begin{aligned} & \sum_{\vec{q}, \vec{s}, \vec{q}', \vec{s}'} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 s}{(2\pi)^4} \frac{1}{q^2} \frac{1}{s^2} \frac{1}{t^2} \frac{1}{r^2} \operatorname{tr} \left\{ \not{q} \left(\delta_{\vec{q}-\vec{q}'} + \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{q}+\vec{q}'} \right) \right. \\ & \Gamma_D^N T_{\mathcal{R}}^B \delta_{\vec{q}'-\vec{s}-\vec{t}} \not{s} \left(\delta_{\vec{s}-\vec{s}'} + \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{s}+\vec{s}'} \right) \Gamma_D^R T_{\mathcal{R}}^C \delta_{\vec{s}'+\vec{t}'-\vec{r}'} \delta^{BC} g_{NR} \\ & \left. \left(\delta_{\vec{t}-\vec{t}'} + \eta^C \alpha_R \delta_{\vec{t}+\vec{t}'} \right) \not{t} \left(\delta_{\vec{r}-\vec{r}'} + \lambda_{\mathcal{R}} \otimes \mathcal{P}_{\frac{1}{2}} \delta_{\vec{r}+\vec{r}'} \right) \Gamma_D^M T_{\mathcal{R}}^A \delta_{\vec{p}-\vec{q}+\vec{r}} \right\}, \end{aligned} \quad (3.24)$$

where $t^\mu = q^\mu - s^\mu$ and $r^\mu = q^\mu - p^\mu = q^\mu$ and also here we omit possible chirality projectors for chiral fermions in even dimensions. Also in this case there are sixteen different diagrams which can be grouped according to the number of twisted fermions. Indeed the presence of a twisted gauge boson introduces a sign (η^C), changes the flux of momenta and adds a factor $(8 - D)/D$ ⁷ with respect to the case of no twist, but does not affect the structure of the traces over Dirac and gauge indices.

We begin with diagrams with zero twisted fermions. These are proportional to

$$\sum_{B,C} \delta_{BC} \operatorname{tr} [T_{\mathcal{R}}^A \{T_{\mathcal{R}}^B, T_{\mathcal{R}}^C\}] = \frac{1}{2} \sum_{B,C} \delta_{BC} \mathcal{A} d^{ABC} \quad (3.25)$$

where \mathcal{A} is the four-dimensional anomaly coefficient. Since spinors in the bulk of a D dimensional space (whatever they are chiral or Dirac with respect to the Γ_D^{D+1} projection) are made of four-dimensional Dirac spinors, the anomaly coefficient \mathcal{A} , along with the tadpole, vanishes⁸.

Diagrams with two twisted fermions can be related to Eq. (3.25) simply by using the commutation property

$$T_{\mathcal{R}}^A \lambda_{\mathcal{R}} = \eta^A \lambda_{\mathcal{R}} T_{\mathcal{R}}^A \quad (3.26)$$

⁷In the case of no twist we have a factor D due to $\Gamma_D^N \Gamma_D^R g_{NR}$, while in presence of a twist this changes to $\Gamma_D^N \Gamma_D^R g_{NR} \alpha_R = 8 - D$.

⁸In fact a Dirac fermion $(\eta, \bar{\chi})^T$ in the complex representation \mathcal{R} is equivalent to Weyl spinors η and χ in representations \mathcal{R} and $\bar{\mathcal{R}}$, respectively, and the system is anomaly-free.

and therefore they also vanish.

We now consider diagrams with one twisted fermion. If q is the twisted momentum the amplitude is proportional to

$$tr \left\{ \not{q} \mathcal{P}_{\frac{1}{2}} \not{s} \not{r} \Gamma_D^M \right\}. \quad (3.27)$$

Also in this case we compute it for $M = i$. We immediately observe that we have reduced to the same situation of the second and the third terms of Eq. (3.22) and, in an analogous way, it can be shown that also this diagram can be non zero only for $D = 6$. If the twisted momentum is s or r , these diagrams differ from the one discussed above simply by a permutation and therefore we obtain the same result.

The diagram with three twisted fermions can be related to Eq. (3.27) by taking into account the commutation property of $\mathcal{P}_{\frac{1}{2}}$

$$\Gamma_D^M \mathcal{P}_{\frac{1}{2}} = \pm \alpha_M \mathcal{P}_{\frac{1}{2}} \Gamma_D^M, \quad (3.28)$$

where the “ $-$ ” sign corresponds to $\mathcal{P}'_{\frac{1}{2}}$, as well as the property $\mathcal{P}_{\frac{1}{2}}^2 = 1$. Therefore we conclude that also this one can be non-zero only for $D = 6$.

4 CONCLUSIONS AND OUTLOOK

In orbifold theories with Higgs-gauge unification Higgs fields A_i are internal components of higher-dimensional gauge fields and as such they transform in the vector representation of the tangent space rotation group $SO(d)$. A quadratically divergent mass term for Higgs fields in the bulk is forbidden by the higher-dimensional gauge invariance while the remnant shift symmetry allows for a similar term localized at the orbifold fixed points only through the non-abelian component of a $U(1)$ field strength tadpole. We have obtained the general conditions an orbifold gauge theory must fulfill for the absence of tadpoles localized at fixed points where $U(1)$ factors are left over by the orbifold projection. On the one hand the internal rotation group at an orbifold \mathbb{G}_f -fixed point is defined as the $SO(d)$ subgroup commuting with \mathbb{G}_f . On the other hand the localized tadpoles can only appear as the invariant terms $\epsilon^{ij} F_{ij}$, where ϵ^{ij} is the Levi-Civita connection which transforms covariantly under the rotation group $SO(2) \subseteq SO(d)$. In this way the existence of tadpoles is tied to the possibility that $SO(2)$ be a subgroup factor of the internal rotation group. Stated differently, the rotation group acting on some internal indices being $SO(p)$ ($p > 2$) is enough to guarantee the absence of tadpoles involving the corresponding components. A particularly simple example is provided by T^d/\mathbb{Z}_2 orbifolds where the internal rotation group for all fixed points is $SO(d)$. There the absence of tadpoles is guaranteed for theories with $d > 2$.

In this paper we did not attempt to construct a realistic Higgs-gauge unification model but only to fix the general conditions for the absence of tadpoles and providing general examples where these conditions are fulfilled. A number of problems should be solved

before a realistic model can be drawn. First of all we must consider models with $D > 6$ dimensions. In fact $D = 5$ models were first studied and they generically lead to too low Higgs masses due to the absence of quartic terms in the potential⁹. $D = 6$ models have a quartic coupling from gauge interactions but it was proven that they generically lead to UV sensitivity through the localized tadpoles¹⁰ except for \mathbb{Z}_2 models with only bulk gauge fields. Since bulk fermions are generically required to trigger electroweak symmetry breaking we are thus led to consider models with $D > 6$ dimensions. The trivial examples we presented in this paper predict the existence of d Higgs fields leading to non-minimal models. Of course the conditions that preclude the existence of quadratic divergences for Higgs fields do not forbid the radiative generation of finite $\sim (1/R)^2$ masses, that vanish in the $R \rightarrow \infty$ limit. Some of the above Higgs fields can acquire different masses and even not participate in the electroweak symmetry breaking phenomenon, depending on the models.

Another problem that we are not facing here is the generation of fermion masses. If fermions are localized at orbifold fixed points they can develop Yukawa couplings through Wilson line interactions after the heavy bulk fermions have been integrated out [11, 24]. One could think of localizing bulk fermions similarly to the $D = 5$ case [25] by giving them a bulk mass [26]. Otherwise the bulk fermions acquire a mass dictated by the higher-dimensional bulk gauge coupling $\sim g_D/(\pi R)^{d/2}v$: if this coupling is large (say comparable to the top quark Yukawa coupling) it can trigger an efficient electroweak symmetry breaking. In this case the four-dimensional gauge couplings should be dominated by localized gauge kinetic terms corresponding to the unbroken subgroups left over at the orbifold fixed points.

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⁹One can avoid this shortcoming by introducing an extra scalar but in this case one reintroduces the problem of quadratic divergences or else one must reintroduce supersymmetry [23].

¹⁰A possible way out is given by the possibility that the local tadpoles are globally vanishing although this leads to strong constraints in the bulk fermion spectrum [12].

A Γ -MATRICES AND TRACES IN ARBITRARY DIMENSION

We work with the metric mostly -1 . For the 4D γ -matrices we adopt the following convention:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad (\text{A.1})$$

with $\sigma^\mu = (\mathbf{1}, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\mathbf{1}, -\vec{\sigma})$ and we define γ^5 as:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}. \quad (\text{A.2})$$

In $D = 2n$ dimensions the Γ -matrices Γ_D^M are defined recursively in this way:

$$\begin{aligned} \Gamma_D^M &= \mathbf{1}_{2 \times 2} \otimes \Gamma_{D-2}^M \quad M = \mu, 5, \dots, D-2 \\ \Gamma_D^{D-1} &= i\sigma_1 \otimes \Gamma_{D-2}^{(D-2)+1} \\ \Gamma_D^D &= -i\sigma_2 \otimes \Gamma_{D-2}^{(D-2)+1}. \end{aligned} \quad (\text{A.3})$$

Γ_D^{D+1} is defined by:

$$\Gamma_D^{D+1} = (-1)^{\frac{D-2}{4}} \Gamma_D^0 \Gamma_D^1 \dots \Gamma_D^D = -\sigma_3 \otimes \Gamma_{D-2}^{(D-2)+1}. \quad (\text{A.4})$$

In $D = 2n+1$ dimensions the first $(D-1)$ Γ_D^M coincide with Γ_{D-1}^M , while $\Gamma_D^D = i\Gamma_{D-1}^{(D-1)+1}$. It is straightforward to verify that these Γ -matrices satisfy the correct anticommutation rules.

We can now proceed to the evaluation of the traces of these Γ -matrices. We list here the results in the case of D even. The case with odd dimensions can be recovered simply remembering that the Γ_{2n+1} -matrices coincide with the Γ_{2n} , except for $\Gamma_{2n+1}^{2n+1} = i\Gamma_{2n}^{2n+1}$. Then the traces in the odd case can be obtained from the even one, paying attention to the presence or not of Γ_{2n+1}^{2n+1} . For D even we have:

$$\text{tr}\{\text{odd } \# \text{ of } \Gamma_D^M\} = 0 \quad (\text{A.5})$$

$$\text{tr}\{\Gamma_D^M \Gamma_D^N\} = 2^{\frac{D}{2}} g^{MN} \quad (\text{A.6})$$

$$\text{tr}\{\Gamma_D^M \Gamma_D^N \Gamma_D^P \Gamma_D^Q\} = 2^{\frac{D}{2}} (g^{MN} g^{PQ} - g^{MP} g^{NQ} + g^{MQ} g^{NP}) \quad (\text{A.7})$$

$$\text{tr}\{\Gamma_D^M \Gamma_D^N \Gamma_D^P \Gamma_D^Q \Gamma_D^R \Gamma_D^S\} = 2^{\frac{D}{2}} (g^{MN} g^{PQ} g^{RS} - \dots) \quad (\text{A.8})$$

and so on. If Γ_D^{D+1} is involved, the trace is always zero unless Γ_D^{D+1} is multiplied by D Γ_D^M with all the M different; it is precisely:

$$\text{tr}\{\Gamma_D^{M_1} \dots \Gamma_D^{M_D} \Gamma_D^{D+1}\} = -(-1)^{\frac{D-2}{4}} 2^{\frac{D}{2}} \epsilon^{M_1 \dots M_D} = i(2i)^{\frac{D}{2}} \epsilon^{M_1 \dots M_D}. \quad (\text{A.9})$$

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